Method For Fatigue Life Estimation Using Modified Endurance Limit

Dhananjay Mishra  
Student  
University of petroleum and energy studies Dehradun, India  
nirmaldj@rediffmail.com

Sandeep Sharma  
Technical Head  
Aerosphere, Chandigarh, India  
sandeep@aerosphere.in

Ankit Dhyani  
Team Lead  
Aerosphere, Chandigarh, India  
dhyani225@gmail.com

ABSTRACT
Fatigue is the failure of a component of a structure member subjected to cyclic load of constant or varying amplitudes. The paper presents various factors that affect the fatigue life of a component. The paper also presents the fatigue life calculation of a piston accumulator based on stress life approach. Basic application of piston accumulator include shock absorption and energy storage.

Keywords
Notch sensitivity, quenching, austenitizing, stress concentration factor, surface finish, fillets and grooves.

1. INTRODUCTION
The failure of a component subjected to cyclic stress is fatigue which basically occurs in three phases: crack initiation, Crack propagation, and catastrophic overload failure. The duration of each phase depends on various factors such as fundamental raw material characteristics, orientation of applied stresses, processing history, magnitude etc. Applied stress levels which lead to fatigue failure are significantly small in magnitude than those stress levels which are necessary to cause static failure.

2. APPROACH TO FATIGUE FAILURE IN ANALYSIS AND DESIGN
2.1 Stress Life
R.R.Moore’s high-speed rotating-beam machine is used to determine fatigue strength. In this setup specimens are subjected to pure bending by means of weights. S-N Curve is further obtained by plotting a curve between the stress applied and the number of cycles that the specimens failed under the test. The results should be plotted in semi log or log-log scale.

2.2 Strain Life
Plastic deformation is one of the most important aspects of fatigue failure. Localized regions are very prone to plastic straining which leads tonucleation of fatigue cracks. Therefore, cyclic strain controlled tests can better characterize fatigue behavior of a material than cyclic controlled tests particularly in the low cycle fatigue region or notched members.

3. ENDURANCE LIMIT AND FATIGUE STRENGTH
3.1 Endurance Limit
Certain materials have a fatigue limit or endurance limit which represents a stress level below which the material does not fail and can be cycled infinite number of times. Hence if the applied stress level is less than the endurance limit of the material, the structure is said to have an infinite life.

3.2 Fatigue Strength
Number of stress cycles of a specific character that a material of specimen can withstand before failure is known as fatigue strength. Number of cycles of failure decreases with the increase in stress amplitude. The fatigue strength is also known as fatigue limit and depends on various factors such as temperature, environment, surface finish, heat treatments etc.

4. ENDURANCE LIMIT MODIFYING FACTOR
4.1 Heat Treatment
Residual stresses can be both produced and relieved by using many heat treatment cycles for both ferrous and non-ferrous alloys. The principal source of residual stress occurs in Cooling processes such as quenching from high temperatures, solutionaising or austenitizing treatments. The chief cause of residual stresses is non-uniform cooling rates between surface and core. These surface compressive stresses are of sufficient magnitude to increase the fatigue strength of a material.

4.2 Surface Finish
A surface finishing process changes the fatigue properties of a part by affecting at least one of the surface characteristics: smoothness residual stress level or metallurgical structure. Greater is the surface finish more will be the fatigue life of the component.

4.3 Temperature
Elevated temperatures might weaken the metals while low temperatures may cause some metals to become notch sensitive and fail by brittle fracture under load conditions that would be nominal at normal temperatures. For most of the metals fatigue life decreases with increase in temperature. Decrease in temperature increases the crack nucleation period. Moreover decrease in temperature decreases the observable crack growth.
4.4 Design

Manufactured parts include many changes in cross section such as grooves, fillets, holes and keyways; which result non-uniform distribution of elastic strain and stress. The actual fatigue limit of a notched member is frequently higher than would be expected from the geometric stress concentration factor. Using notch sensitivity factor results in a fatigue stress concentration factor that is determined from

\[ K_f = q (K_t - 1) + 1 \]

\( K_f \) = fatigue stress concentration factor for direct tension or bending; \( q \) = notch sensitivity
\( K_t \) = geometric stress concentration factor for direct tension or bending

5. Problem Statement

5.1 Fatigue Analysis for Piston Accumulator

In this section we analyze the Operating Life of Accumulator by calculating the life of accumulator.

We know that accumulator is being used to work in pressure conditions viz. \( p_1 = 300 \) bar and \( p_2 = 350 \) bar. Preassembly pressure is 145 bar. We have to derive impact/cycles lifetime (the same can be changed into years and months).

5.1.1 Provided Input for Flange

Let’s assume the following data for the piston accumulator

Maximum Allowable Pressure= 350 bar
Pressure Cycle invariable amplitude \( \Delta P = 50 \) bar
Design Temperature = -25 ...100 Celsius

Chosen internal Diameter of Cylinder \( D_i = 150 \) mm
Chosen cylinder wall thickness \( t = 12.5 \) mm
Max Tolerance under wall thickness \( \delta = 0.9 \) mm
Chosen end flange thickness \( e = 60 \) mm
External thread diameter \( D_u = 154 \) mm
Thread Pitch \( L_p = 2 \) mm
Internal diameter of thread
\[ D_p = D_u - 1.0825 \times (L_p) = 151.835 \text{ mm} \]
Central Diameter of thread
\[ E_{min} = D_u - 0.6495 \times (L_p) = 152.701 \text{ mm} \]

Chosen Thread length = 39.7 mm
Maximum Opening in end flange \( d_c = 0.6D_i \)
Chosen Width of Seal groove \( e_t = 8.1 \) mm

5.1.2 Load Cases

Assuming the following stress cycles acting on the piston Accumulator

Working Pressure for the piston accumulator is defined as
\( \sigma_{max} = 415 \) bar
Pre assembly pressure
\( \sigma_{min} = 145 \) bar

The calculations below is for estimating the fatigue life of the piston accumulator

5.1.3 Endurance Strength Calculation

Fatigue Strength Fraction, \( f := 0.67 \)
Tensile Strength \( s_{ut} = 350 \) MPa
Yield strength \( s_y = 280 \) MPa
Endurance Limit \( s_{e1} = 0.5 \times s_{ut} \)

As Per Marine Equation

\[ S_e = \left( S_{e1} \right)^{\frac{1}{k_a k_b k_c k_d k_e k_f}} \]

Where:
\( S_{e1} \) = endurance limit
\( S_e \) = modified endurance limit
Where, \( k_a = \) surface factor
\( k_b = \) size factor
\( K_c = \) Loading factor
\( K_t = \) Temperature factor
\( K_e = \) Reliability factor
\( K_f = \) Miscellaneous factor

Note: Notch concentration and notch sensitivity are ignored

5.1.4 Calculation of Surface factor \( k_a \)
(Referenced to Metal Engineering Design ASME handbook)
For a machined part

\[ a = 4.51 \]
\[ b = 0.265 \]
As defined earlier the ultimate strength

\[ S_u = 350 \text{ Mpa} = a^* (s_u)^b \]

\[ K_a = 0.955 \]

5.1.5 **Size Factor \( k_b \)**

Given internal diameter

\[ d_0 = 150 \text{ mm} \]

Size factor is calculated as;

\[ k_b = \begin{cases} 
-0.107, & \text{if } 0.002 \leq d \leq 0.0508 \\
-0.157, & \text{if } 0.0508 < d \leq 0.254 \\
-0.107, & \text{if } 0.254 < d \leq 0.51 \\
-0.157, & \text{if } 0.51 < d \leq 2.51 \\
1, & \text{otherwise} 
\end{cases} \]

\[ K_b = 0.688 \]

5.1.6 **Loading Factor \( k_c \)**

\[ k_c = \begin{cases} 
0.85, & \text{bending} \\
0.59, & \text{axial} \\
0.5, & \text{torsion} \]

\[ K_c = 0.85 \]

5.1.7 **Reliability Factor \( k_e \)**

Assuming reliability factor of 95%

\[ Z_a = 1.645 \]

\[ K_a = 1-0.08Z_a \]

\[ K_e = 0.868 \]

5.1.8 **Miscellaneous Effect Factor \( k_f \)**

\[ K_f = 1 \]

Note: Notch Concentration and Notch Sensitivity are ignored

So endurance limit is now defined as

\[ S_e = (S_{e1})^{k_d}k_bk_ck_dk_e^2k_f \]

\[ S_e = 86.518 \text{ MPa} \]

**Load Input Given**

Working Pressure for the piston accumulator is defined as

\[ \sigma_{\max} = 415 \text{ bar} \]

\[ \sigma_{\min} = 145 \text{ bar} \]

Stress Amplitude is defined as

\[ \sigma_a = \frac{(\sigma_{\max} - \sigma_{\min})}{2} \]

\[ \sigma_a = 135 \text{ Mpa} \]

Mean stress \( \sigma_m = 28 \text{ MPa} \)

**To calculate no of cycles**

Calculation of parameters

\[ a = \frac{(fS_{ut})^2}{S_e} \]

\[ a = 635.594 \text{ Mpa} \]

\[ b = 0.144 \]

Failure criteria as selected is modified Goodman line;

\[ S_{fg} = \frac{1.5\sigma_a}{1 - \frac{1.5\sigma_m}{S_y}} \]

\[ S_{fg} = 23.824 \text{ Mpa} \]

\[ N = (S_{fg}/a)^{1/b} \]

\[ N = 7.591 \times 10^9 \text{ cycles} \]

6. **TABLES AND FIGURES**
Fig 1: S-N CURVE OF A COMPONENT

Fig 2: CURVE FOR FATIGUE STRENGTH VS NO. OF CYCLES
(taken from metals engineering design ASME handbook)

Fig 3: FATIGUE FRACTION vs. TENSILE STRENGTH

<table>
<thead>
<tr>
<th>TEMP(°C)</th>
<th>Sf/SRT</th>
<th>TEMP(°F)</th>
<th>Sf/SRT</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1.000</td>
<td>70</td>
<td>1.000</td>
</tr>
<tr>
<td>50</td>
<td>1.010</td>
<td>100</td>
<td>1.008</td>
</tr>
<tr>
<td>100</td>
<td>1.020</td>
<td>200</td>
<td>1.020</td>
</tr>
<tr>
<td>150</td>
<td>1.025</td>
<td>300</td>
<td>1.024</td>
</tr>
<tr>
<td>200</td>
<td>1.020</td>
<td>400</td>
<td>1.018</td>
</tr>
<tr>
<td>250</td>
<td>1.000</td>
<td>500</td>
<td>0.975</td>
</tr>
<tr>
<td>300</td>
<td>0.975</td>
<td>600</td>
<td>0.963</td>
</tr>
<tr>
<td>350</td>
<td>0.943</td>
<td>700</td>
<td>0.927</td>
</tr>
<tr>
<td>400</td>
<td>0.900</td>
<td>800</td>
<td>0.872</td>
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<tr>
<td>450</td>
<td>0.843</td>
<td>900</td>
<td>0.797</td>
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<tr>
<td>500</td>
<td>0.768</td>
<td>1000</td>
<td>0.698</td>
</tr>
<tr>
<td>550</td>
<td>0.672</td>
<td>1100</td>
<td>0.567</td>
</tr>
<tr>
<td>600</td>
<td>0.579</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig 4: EXPERIMENTAL DATA FOR CALCULATION OF TEMPERATURE FACTOR K₀
(Taken from mechanical engineer design)
### Table 1: Experimental Data for Calculation of Reliability Factor

<table>
<thead>
<tr>
<th>Reliability %</th>
<th>Transformation Variate $Z_a$</th>
<th>Reliability factor $k_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0</td>
<td>1.000</td>
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<td>90</td>
<td>1.288</td>
<td>0.897</td>
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<td>95</td>
<td>1.645</td>
<td>0.868</td>
</tr>
<tr>
<td>99</td>
<td>2.326</td>
<td>0.814</td>
</tr>
<tr>
<td>99.9</td>
<td>2.091</td>
<td>0.753</td>
</tr>
<tr>
<td>99.99</td>
<td>3.719</td>
<td>0.702</td>
</tr>
<tr>
<td>99.999</td>
<td>4.265</td>
<td>0.659</td>
</tr>
</tbody>
</table>

Fig 5: EXPERIMENTAL DATA FOR CALCULATION OF RELIABILITY FACTOR.  
(Taken from mechanical engineer design)

Fig 6: CROSS SECTION VIEW OF A PISTION TYPE ACCUMULATOR

### 6. FUTURE WORK

- Selection of failure criteria on the basis of criticality of component.
- Study of effect of material properties and fatigue strength.
- Fatigue life calculation using different fatigue criteria.
- Study of damage growth curve and residue strength curve.
7. REFERENCES


