

Sizing Procedure for Shear Resistant Beams

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ABSTRACT

Beams being a vital part of any structural design in aircrafts, vehicles, submarines, ships or even civil engineering structures, carry different types of loads. Loads can be axial, bending, shear or torsional in case of beam boxes. Since in today's profit driven industry - a lot of emphasis is given on cost and reduction in time required for designing while maintaining quality of the design, so modern methods for designing and testing are essential. By help of a simple example, this paper tries to build upon the conventional sizing procedures and theory of shear resistant beams used in industry to include in it various changes and additions. Firstly space constraints, loading conditions and operating environments are identified. After selecting material based on the above, sizing procedure for flange, stiffener and finally web is shown using iteration method done in Microsoft Excel. Conditions for selecting minimum critical dimensions of all three are discussed. Workflow formulation is also done so the desired result is obtained in least possible time without the need to go through some parts regularly. Basis of follow up research papers to include Ansys analysis and the development of sizing procedure for Hybrid Diagonal Tension Beams has been prepared.

Keywords

Shear Resistant Panels, Sizing of Stiffeners,

1. INTRODUCTION

Shear resistant beams though used in a very limited amount in airframe structures still has vital uses in aircrafts as the floor supports cannot be made by hybrid diagonal tension beams as the easily buckle and also in spars where buckling has to be checked to maintain aerodynamic design of the wing. Furthermore, for weight optimising sizing procedures for hybrid diagonal tension beams, concepts of sizing for both shear resistant and diagonal resistant beams should be clear. Also shear resistant beams are used in civil and automotive engineering in areas where buckling cannot be permitted.

So, this research paper aims to carry forward the sizing procedures established by Niu (1999) by changing the all three, namely web, flange and stiffener cross sections and highlight how we can use this method in weight optimization.

2. METHODOLOGY

2.1 Formulation of Problem

The process of sizing the shear resistant beams involve first of all identifying the loading conditions and dimensions of beam required. Once obtained the working environment, temperature variations, beam's purpose and whether it is being designed to withstand cyclic loads is also determined. This helps in selecting the material for beam as certain materials resist creep where as some other material may be used for higher fatigue resistance.

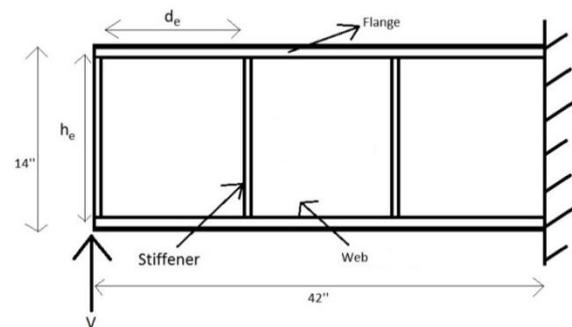


Figure-1 Beam to be used in Example

In our example we use a beam as shown in figure 1 whose length required is 42" and height required is 14" which would take up vertical shear load 'V' of 10,000 lb. The material would be Al 7075 T6 bare sheet for web and Al 7075 Extrusion for flanges and stiffeners. The first step is to determine the shape of the beam to be used. For standard I-beam cross section, the CAD model shown in figure 2 describes how an I-beam is made of 2 flanges and a single rectangular plate web with the stiffeners-attached to web and flange through rivets.

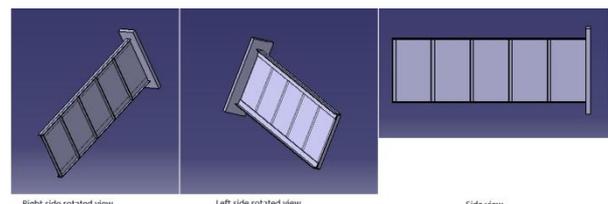


Figure-2 Pictures of CAD model showing the beam

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	Method for Sizing of Shear resistant beam																	
2	Material for Web :- Al 7075-T6 bare sheet Material for caps and stiffeners:- Al 7075-T6 Extrusion																	
3	Properties (Fixed)												Figure-4 (For Elastic Range Only)			Typically available T-sections (Figure-5)		
4	Iteration	V (Force in lb)	E (psi)	F _{cy} (psi)	ρ (lb/in ³)	h (in)	L (in)	M _{max} (lb in)	h _e (inch) Temporary	q (lb/in) temp	t (inch)	d _e (inch)	A _{cap} (in ²) req approx.	t _w (in)	Y _t (in)	h _e (inch) Actual (heA)	A _{cap} (in ²)	l _{xx} (in ⁴)
5	1	10000	10500000	74000	0.102	14	42	420000	12	833.333	0.064	5	0.472973	0.156	0.37	13.26	0.437	0.0725
6	2	10000	10500000	74000	0.102	14	42	420000	12	833.333	0.072	6	0.472973	0.156	0.37	13.26	0.437	0.0725
7	3	10000	10500000	74000	0.102	14	42	420000	12	833.333	0.081	7	0.472973	0.156	0.37	13.26	0.437	0.0725
8	4	10000	10500000	74000	0.102	14	42	420000	12	833.333	0.091	8	0.472973	0.156	0.37	13.26	0.437	0.0725
9	5	10000	10500000	74000	0.102	14	42	420000	12	833.333	0.102	10	0.472973	0.156	0.37	13.26	0.437	0.0725
10	6	10000	10500000	74000	0.102	14	42	420000	12	833.333	0.125	17	0.472973	0.156	0.37	13.26	0.437	0.0725
11	7	10000	10500000	74000	0.102	14	42	420000	12	833.333	0.156	28	0.472973	0.156	0.37	13.26	0.437	0.0725

(a)

	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH
1	Typically available Stiffener Sections ; Only selected sections from next two sheets are shown															
2	Section side attached with Web shown in bold ; I _u Actual calculated for attached side only															
3	Figure-6			Table-1			Figure-9									
4	Min. A _{cap} req (in ²) exact(Acc)	h _{eA} /d _e	I _u /h _{eA} t ³	I _u (in ⁴) req.	Min. t _u (in) required	A (in)	B (in)	t _u (in)	A _u (in ²)	Y _s (in)	X _s (in)	I _x (in ⁴)	I _y (in ⁴)	I _u (in ⁴) Actual	ELS / ULS Sheet	K _s
5	0.4280298	2.652	1.95	0.006778257	0.05	0.75	0.75	0.0625	0.089844	0.199	0.199	0.004	0.004	0.007558	Row 2 (ELS)	5.3
6	0.4280298	2.21	1.6	0.00791883	0.06	0.75	0.5	0.063	0.074781	0.12	0.243	0.0014	0.0041	0.008516	Row 2 (ULS)	5.6
7	0.4280298	1.8943	1.4	0.009865671	0.06	0.875	0.625	0.063	0.090531	0.15	0.269	0.0029	0.0069	0.013451	Row 4 (ULS)	5.8
8	0.4280298	1.6575	1	0.009992351	0.07	0.75	0.75	0.09375	0.131836	0.214	0.214	0.0063	0.0063	0.012338	Row 3 (ELS)	6.3
9	0.4280298	1.326	0.5	0.007035809	0.07	0.75	0.75	0.09375	0.131836	0.214	0.214	0.0063	0.0063	0.012338	Row 3 (ELS)	6.8
10	0.4280298	0.78	0.1	0.002589844	0.08	0.75	0.75	0.09375	0.131836	0.214	0.214	0.0063	0.0063	0.012338	Row 3 (ELS)	7
11	0.4280298	0.4736	0.04	0.002013619	0.08	0.75	0.75	0.09375	0.131836	0.214	0.214	0.0063	0.0063	0.012338	Row 3 (ELS)	5.8

(b)

	AI	AJ	AK	AL	AM	AN	AO	AP	AQ	AR	AS	AT
1	Mass of Stiffeners											
2												
3												
4	F _{s,cr} (psi)	q (lb/in)	f _s (psi)	F _{s,cr} > f _s	A _{cap} > Acc	t _u > t _u required	I _u actual > I _u req.	Volume of one Stiffener (in ³)	Mass of one stiffener (lb)	Number of stiffeners required	Next rounded of value	Mass of all stiffeners (lbm)
5	9117.696		754.147813	11783.56 not success	success	success	success	1.191328125	0.121515469	7.4	8	0.97212375
6	8467.2		754.147813	10474.28 not success	success	success	success	0.99159606	0.101142798	6	6	0.606856789
7	8154.385714		754.147813	9310.467 not success	success	success	success	1.20044106	0.122444988	5	5	0.612224941
8	8559.189844		754.147813	8287.339 Success	success	success	success	1.748144531	0.178310742	4.25	5	0.891553711
9	7428.456		754.147813	7393.606 Success	success	success	success	1.748144531	0.178310742	3.2	4	0.713242969
10	3973.83218		754.147813	6033.183 not success	success	success	success	1.748144531	0.178310742	1.47058824	2	0.356621484
11	1890.385714		754.147813	4834.281 not success	success	success	success	1.748144531	0.178310742	0.5	1	0.178310742

(c)

	AU	AV	AW	AX	AY
1					
2					
3	Mass of Web				
4	Volume of Web (in ³)	Mass of Web (lbm)	Mass of stiffener + Web (lbm)	Manufacturable de (inch)	Usable?
5	0.886016	0.0903736	1.06249738	5.25	not usable
6	0.996768	0.1016703	0.70852712	7	not usable
7	1.121364	0.1143791	0.72660407	8.4	not usable
8	1.259804	0.1285	1.02005372	8.4	usable
9	1.412088	0.144033	0.85727594	10.5	usable
10	1.7305	0.176511	0.53313248	21	not usable
11	2.159664	0.2202857	0.39859647	42	not usable

(d)

Figure-3 MS Excel Worksheet for Solution for given Problem

2.2 Sizing

2.2.1 Sizing of Flanges

The flange would consist of a T-section beam whose centroid location ‘Y’, should be approximated for starting iteration. Let it be 1 inch for iteration 1. So approximate ‘ h_e ’ which is distance between centroids of top and bottom flange could be taken as 12 inch. So, ‘ q ’ (shear flow) can be calculated by formula $q = V/h_e$. For ‘ $h_e = 12$ ’, q comes out to be 833.33. For this value using Figure-4 different values of ‘ t ’ and ‘ d_e ’ will be determined on which further calculations and iterations of web and stiffener design would be based.

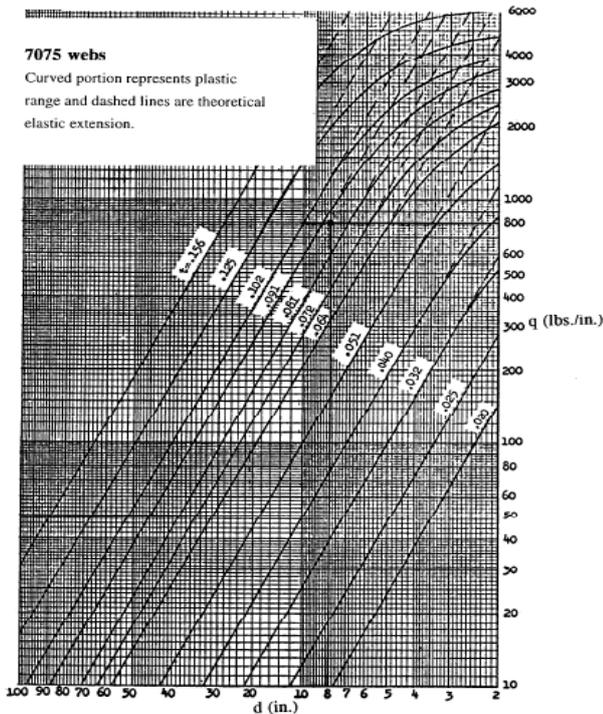


Figure-4 Web thickness and Stiffener spacing curves for Al 7075

Web Thickness (t)	Thickness of Attached Leg of Stiffener (t_{st})
0.02	0.03
0.025	0.03
0.032	0.04
0.036	0.04
0.04	0.04
0.05	0.05
0.063	0.05
0.071	0.06
0.08	0.06
0.09	0.07
0.1	0.07
0.125	0.08
0.156	0.08
>0.156	0.6t

Table-1 Web thickness v/s Thickness of Attached Leg of Stiffener

Using approximate value of ‘ h_e ’ approximate minimum required area of flange, to resist the stresses induced in flanges due to shear loading, given by $A_{CAP} = M/h_e * f_{CAP}$ is calculated, where f_{CAP} is maximum shear stress which AL 7075 Extrusion can carry and ‘M’ is maximum moment on

flange given by 'V*L'. Based on values of A_{CAP} , 'Y' and already existing and available list of T- sections the section shown in Figure-5 below is selected. Now using actual $h_e = 14.2*Y$, calculate actual h_e (h_eA) which would be repeatedly used in further calculations. Now exact minimum required A_{CAP} is calculated immediately and compared with actual A_{CAP} of section in figure as, if the latter is not greater than the former, iterations would have to be repeated till this condition is satisfied. Satisfying this condition here rather than at last saves a lot of time for undergoing lengthy iterations.

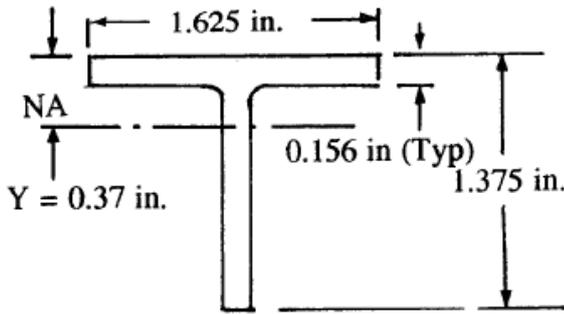


Figure-5 A Typical T-section used in this example as Flange

2.2.2 Sizing of Stiffener

Calculate h_eA/d_e and then using Figure-6 obtain the value for $I_u/h_eA.t^3$. From this value obtained from graph, and knowing the values of 't' and ' h_eA ', we can find the value of minimum I_u required where I_u is the moment of inertia of stiffener about the axis located at start of the stiffener parallel to the web. This condition is set to make sure that stiffener does not buckle itself while carrying out its job of prevention of buckling in web.

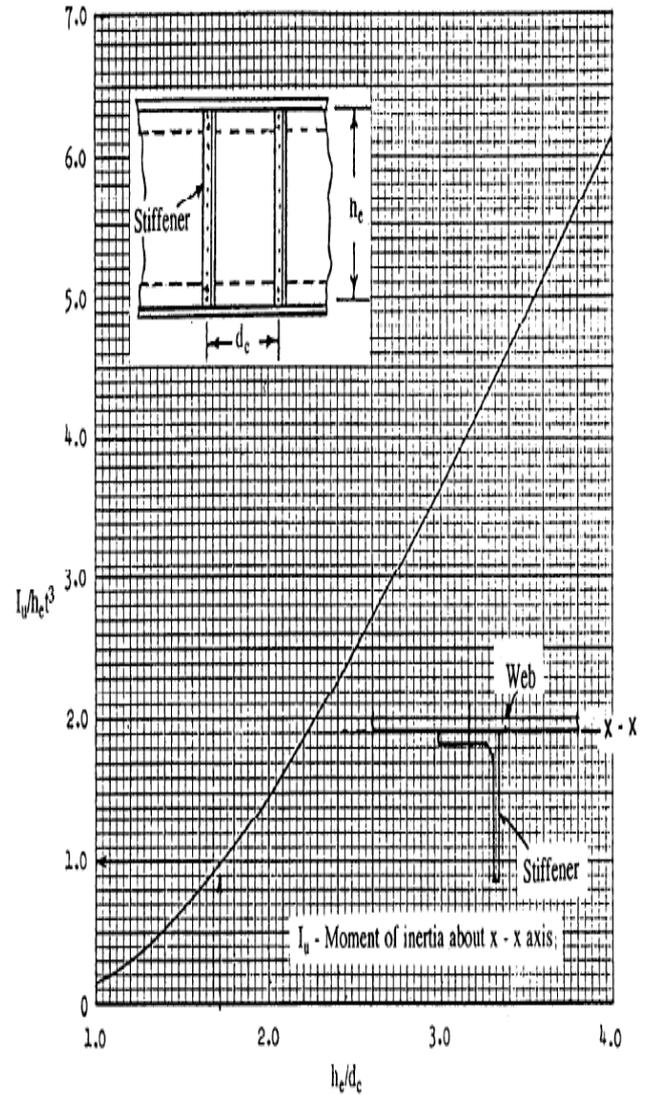


Figure-6 Graph for I_u/h_e*t^3

Typically available 'Equal Leg Stiffener' and 'Unequal Leg Stiffener' sections dimensionally defined as shown in Figure-7 are catalogued in separate Excel sheets shown in Figure-8. Since the ultimate aim is to reduce weight, so stiffeners need to be selected keeping in mind the same basis.

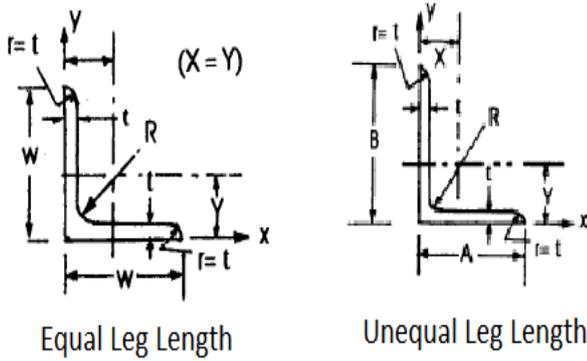


Figure-7 Equal and Unequal Leg Stiffener Sections

1. Length of leg of stiffener attached to web should agree by Table-1.
2. I_u of stiffener should be greater than minimum I_u required as calculated before.
3. Area of cross section of stiffener should be as least as possible to reduce weight.
4. If values of all of the above three conditions are nearly equal for two or more stiffeners then moment of inertia about the axis perpendicular to the above mentioned axis is compared. The design having lower moment of inertia is chosen as buckling load about a given axis is proportional to moment of inertia; since the stiffener if at all buckles we don't want it to buckle about the axis parallel to axial length of web but perpendicular to it.

The minimum required conditions are:-

Equal Leg Stiffener Calculations							Unequal Leg Stiffener Calculations										
A	B	C	D	E	F		A	B	C	D	E	F	G	H	I	J	
w (in)	t (in)	A (in ²)	Y (in)	I_x (in ⁴)	I_u		A (in)	B (in)	t (in)	A (in ²)	Y (in)	X (in)	I_x (in ⁴)	I_y (in ⁴)	I_{uc} (in ⁴)	I_{vy} (in ⁴)	
1	0.75	0.0625	0.089	0.199	0.004	0.007524489	2	0.75	0.5	0.063	0.0739	0.12	0.243	0.0014	0.0041	0.002464	0.008464
2	0.75	0.09375	0.132	0.214	0.006	0.012045072	3	0.75	0.625	0.063	0.0818	0.162	0.222	0.0027	0.0044	0.004847	0.008431
3	0.75	0.125	0.171	0.227	0.008	0.016811459	4	0.875	0.625	0.063	0.0922	0.15	0.269	0.0029	0.0069	0.004975	0.013572
4	1	0.0625	0.122	0.271	0.012	0.020959802	5	0.875	0.75	0.063	0.1	0.19	0.25	0.0049	0.0073	0.00851	0.01355
5	1	0.09375	0.178	0.276	0.016	0.029559328	6	1	0.625	0.063	0.1	0.14	0.32	0.0029	0.01	0.00486	0.02024
6	1	0.125	0.234	0.29	0.021	0.0406794	7	1	0.625	0.125	0.184	0.16	0.344	0.005	0.017	0.00971	0.038774
7	1	0.1875	0.339	0.314	0.029	0.062424044	8	1	0.75	0.063	0.108	0.178	0.299	0.0051	0.0106	0.008522	0.020255
8	1.25	0.09375	0.23	0.34	0.033	0.059588	9	1	0.75	0.125	0.2	0.199	0.322	0.0085	0.0182	0.01652	0.038937
9	1.25	0.125	0.3	0.35	0.042	0.07875	10	1.25	0.75	0.063	0.124	0.159	0.403	0.0054	0.0157	0.008535	0.039839
10	1.25	0.1875	0.43	0.37	0.059	0.117867	11	1.25	0.75	0.094	0.179	0.17	0.416	0.0074	0.0276	0.012573	0.058577
11	1.25	0.25	0.56	0.4	0.074	0.1636	12	1.25	0.75	0.125	0.231	0.181	0.427	0.0092	0.0346	0.016768	0.076718
12	1.5	0.125	0.36	0.41	0.074	0.134516	13	1.25	1	0.063	0.139	0.239	0.361	0.0124	0.0217	0.02034	0.039815
13	1.5	0.1875	0.53	0.44	0.107	0.209608	14	1.25	1	0.094	0.202	0.25	0.373	0.0171	0.0306	0.025725	0.058704
14	1.5	0.25	0.69	0.46	0.135	0.281004	15	1.5	0.75	0.094	0.204	0.156	0.522	0.0077	0.0463	0.012665	0.01387
15	1.75	0.09375	0.32	0.47	0.096	0.166688	16	1.5	0.75	0.125	0.254	0.167	0.534	0.0095	0.0584	0.016969	0.133581
16	1.75	0.125	0.42	0.47	0.121	0.213778	17	1.5	0	0.094	0.228	0.228	0.472	0.0182	0.0512	0.030052	0.101395
17	1.75	0.1875	0.62	0.5	0.174	0.329	18	1.5	0	0.125	0.295	0.239	0.485	0.0225	0.0547	0.035451	0.134091
18	1.75	0.25	0.81	0.52	0.223	0.442024	19	1.5	0	0.156	0.351	0.249	0.495	0.0265	0.0772	0.048882	0.165554
19	2	0.125	0.49	0.53	0.18	0.317641	20	1.5	1.25	0.094	0.251	0.31	0.433	0.0347	0.055	0.058821	0.10206
20	2	0.1875	0.72	0.56	0.27	0.495792	21	1.75	1	0.125	0.327	0.222	0.591	0.0235	0.0958	0.035716	0.214015
21	2	0.25	0.94	0.58	0.34	0.656216	22	1.75	1.25	0.125	0.358	0.3	0.545	0.0465	0.1078	0.07872	0.214135
22	2	0.3125	1.16	0.61	0.41	0.841636	23	1.75	1.5	0.125	0.389	0.383	0.506	0.0775	0.1144	0.134662	0.213998

Figure-8 Catalogue of Typically available Equal and Unequal Leg Stiffeners

2.2.3 Sizing of Web

Shear buckling coefficient 'K_s' is found using Figure-9 with condition ④ as shown in graph. Now calculating maximum critical shear stress for web of these dimensions can carry by

using formula $F_{s,cr} = K_s \cdot E \cdot (t/d_c)^2$ where E is young's modulus which is 10500000 psi.

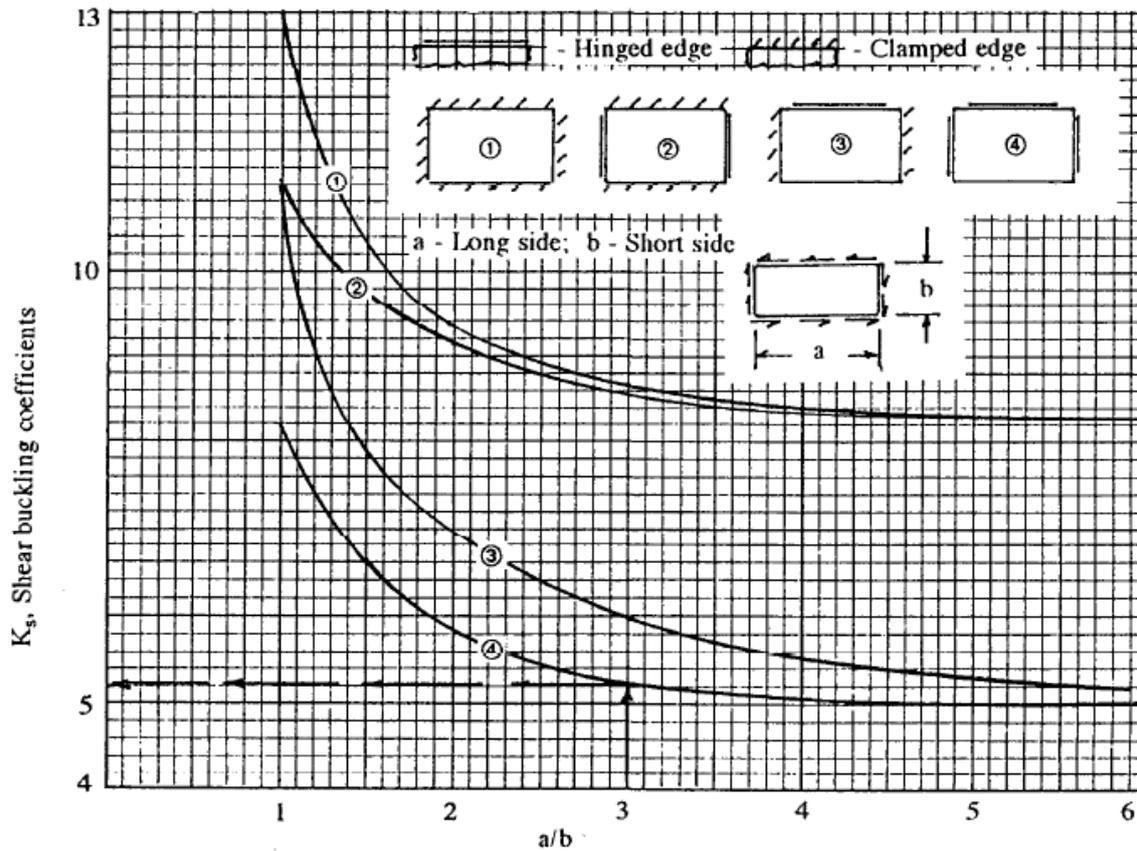


Figure-9 Graph to Calculate value of Shear Buckling Coefficient

After calculating this actual shear flow given by ' $q=V/h_eA$ ' which is further used to calculate shear flow the webs would actually carry (' f_s ') based on the dimensions just designed. Since, ' $f_s = VQ/It$ ' and ' $Qh_eA/I = 1$ ', therefore calculate ' $f_s = q/t$ ' and compare it with ' $F_{s,cr}$ '; former should be always smaller than the latter.

2.3 Finalizing Design

The design could be used if it satisfies the following four conditions :-

1. $F_{s,cr} > f_s$
2. $A_{cap} > \text{Actual Flange Area (A)}$
3. $t_u > t_u \text{ required}$
4. $I_u > I_u \text{ required}$

Further to select one design from shortlisted candidates from above criterion, calculate, for a particular design:-

1. The volume of one Stiffener using relation ' $\text{Volume} = A_u * h_eA$ ',
2. Number of Stiffeners that would be required for a particular design given by $= (42/d_e) - 1$. For ease of manufacture this value is rounded up to next integer and then used,
3. The volume of Web given by $t*(14-t_w)$, where t_w is thickness of flange panel,
4. Mass of single Stiffener,

5. Mass of all Stiffeners,
6. Mass of Web,
7. Mass of all stiffeners + Mass of Web

Diagonal tension that would be carried for a specific load also depends on the distance between stiffeners for a constant height of web. As it is clear from the equivalent truss diagram for pure diagonal tension webs more is the distance between two stiffeners less will be the diagonal tension stress and more stress would be on shear restraint part for hybrid design.

So small distance on one hand is desirable but can on other hand increase both weight and cost of the structure. So, an optimum distance should be maintained between stiffeners depending on the loading conditions and location where the structure is to be used.

3. RESULT

After this sizing procedure, the result obtained as shown in Figure-3 (d), shows that out of seven iterations with variable thickness of web, in five iterations when the stiffener spacing and stiffener moment of inertia for least possible weight is chosen then the criterion of ' $F_{s,cr} > f_s$ ' is not satisfied. Thus they cannot be used for the particular example.

From other two iterations which satisfy all the conditions, the one giving least possible mass of 0.857 lbm for web + stiffener can be chosen since flange is common for all designs so it's mass does not affect the choice.

4. CONCLUSION

The method described in this paper started from analysing the methods formulated by NACA for sizing of beams which were cited by Niu (1999) which are still followed by aircraft manufacturers worldwide. Small parts of the method although same, have some modifications to reduce time for designing the beams utilizing the modern tools like Microsoft Excel. The method used here is optimized both for time saving and weight saving as different sizes of flange, web and stiffener is simultaneously analysed for given space and loading constraints.

The method for sizing of Shear Resistant Beams would be further utilized along with method to size pure diagonal tension webs to analyze the sizing procedure for hybrid diagonal tension webs whose theory has been combined by

Niu (1999). Static Structural analysis for the same example in Ansys would soon follow whose result if within low percentage error range would be able to support this method's usage.

5. REFERENCES

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